A nestable, multigrid-friendly grid on a sphere for global spectral models based on Clenshaw-Curtis quadrature

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Extended Abstract

1. Introduction

Global spectral models typically adopt Gaussian quadrature in performing spherical harmonics transforms, with the consequence of the nodes (grid-points) placed on irregularly-spaced Gaussian latitudes. As we move into very high resolutions, it becomes necessary to accurately represent the nature's nonhydrostatic aspects into the model. Semi-implicit time-stepping applied to nonhydrostatic equations necessitates solution of non-constant coefficient Helmholtz-type problem, which is difficult to solve efficiently by the current massively parallel HPC architecture. A promising approach with demonstrably high parallel efficiency to this type of problem is the multigrid, which exploits the hierarchy of grids to accelerate iterative elliptic solvers. However, a multigrid approach is difficult to implement on the current global spectral models because the Gaussian grids do not nest.

In this study we propose a new, nestable grid on a sphere which would allow straight multigrid implementation. The proposed grid and quadrature rules are implemented on a shallow-water equations (SWE) model and a three-dimensional hydrostatic primitive equations (HPE) model. Detailed description of this work can be found in our recent publication [1].

2. Grid and Quadrature formulation

In the proposed scheme, numerical integration in the meridional direction is performed using a variant of Clenshaw-Curtis-type quadrature (Fejér's second rule) [2] instead of the conventional Gauss-Legendre rule. With this quadrature rule, the nodes are aligned on colatitudes $\theta_j = \frac{j\pi}{J+1}$ $(j = 1, \cdots, J)$, meaning that the grid does not include that poles and the latitudinal grid points are equidistant. This grid is nestable since the grid points for $J/2$ nodes can be constructed by skipping every other grid of the nodes for J-point rule starting from j=2. One shortcoming of this grid is that $J \geq 2N+1$ meridional nodes are required to ensure exact transform for the truncation total wavenumber of N, unlike the Gaussian grid which requires only $(2N-1)/2$ nodes.

3. Numerical orthonormality and aliasing errors on nonlinear terms

The proposed grid and quadrature are implemented on the code of the operational global model of Japan Meteorological Agency (JMA-GSM) and tested for numerical orthonormality of the associated Legendre functions. The orthonormality is found to be satisfied up to machine precision as long as the condition $J \geq 2N+1$ is met. The proposed quadrature formally incurs aliasing errors for nonlinear terms of quadratic or higher orders, but the relative errors caused by the aliasing are found to be small (at most $O(10^{-3})$ in practice.

4. Test case results

An SWE model and an HPE model based on JMA-GSM are adapted to use the proposed Clenshaw-Curtis grid and are compared with their original Gaussian-grid versions within the framework of idealized test cases ([3,4] for SWE and [5] for HPE) with various horizontal resolutions up to Tc479 (Δx～20 km). The integration results from the two versions of the models are confirmed to be nearly identical for any of the investigated test cases.

5. Future direction: Multigrid-based grid-spectral hybrid model

The proposed grid can be adapted to take a structured form (e.g., as in Fig.1). We postulate that employing the pseudo-spectral multigrid method will foster smooth and gradual transition from spectral to grid-based modelling since the pseudo-spectral horizontal derivatives can be readily replaced by local, stencil-based horizontal derivatives. Given that grid-based elliptic solvers tend to be less efficient at larger scale, as grid/spectral hybrid approach, where a grid-based multigrid method with shallow layers is combined with a spectral elliptic solver used only at the coarsest grid with moderate resolution, would be a reasonable strategy that compromises the need to avoid global inter-node communications and to maintain high accuracy and fast convergence rate.

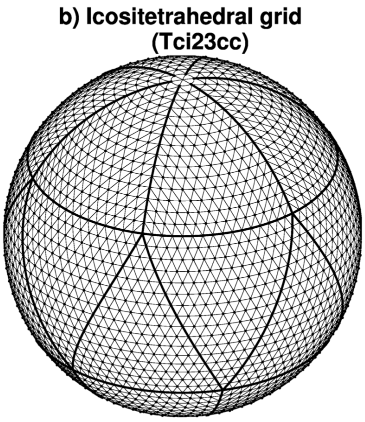

Fig1:Icositetrahedral (24-face polyhedral) Clenshaw-Curtis grid.

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